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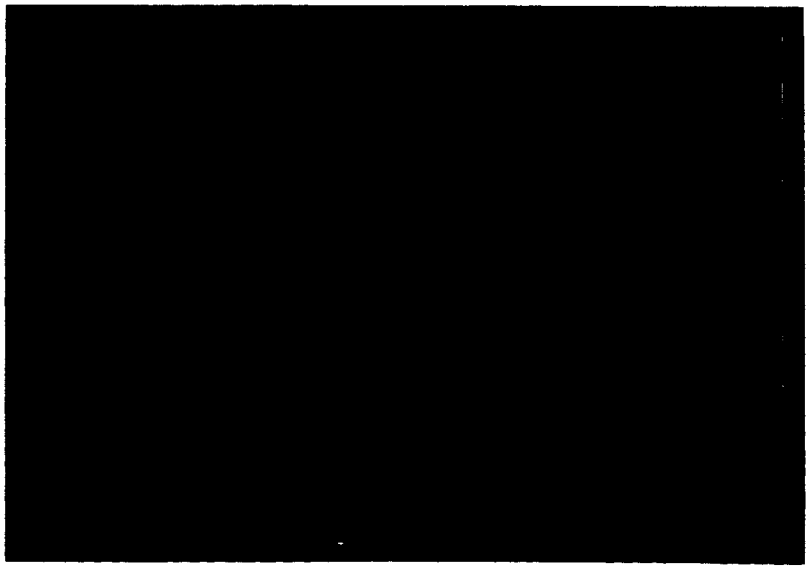

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STATIC AND DYNAMIC FORCES OF PARTIAL ARC SELF-ACTING GAS JOURNAL BEARINGS AT MODERATE COMPRESSIBILITY NUMBERS

by

R.J. Wernick & C.H.T. Pan

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Author(s)

Approved by

Approved by

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I. INTRODUCTION

Partial arc self-acting bearing pads are often used to overcome rotor-bearing instability. In such applications, depending on the specific design requirements, the bearing may be constructed either of a single fixed pad, of several fixed pads (multi-lobe bearing), or of several pivoted or flexibly supported shoes. The vital design information consists of both the static and time dependent fluid film forces of each pad, which ultimately determines the load carrying capacity of the bearing and the stability characteristics of the rotor-bearing system.

An important application of the partial-arc type bearings is the self-acting gas journal bearings operating in a high ambient pressure. Here the lateral unit load on the journal bearings is extremely small so that *Half-Frequency Whirl instability is likely to occur for plain cylindrical journal bearings. Under this condition, the bearing compressibility number, Λ, is always very small, thus, it is convenient to solve the lubrication problem by an expansion of the fluid film pressure in terms of a power series in Λ [6]. The first order effect of this expansion actually gives the same equation applicable to an incompressible lubricant. The higher order effects are concerned with equations of increasing complexity, but are nevertheless all linear.

Solution of these equations is no simple matter because of the variable coefficients. Tanner [1] has applied the method of Galerkin to solve the steady state Reynolds equation for a variety of bearing configurations. The present paper extends this method to consider both time-dependence and higher order effects of Λ. Under the assumptions made herein, the results also may be applied to the calculation of critical speed, rotor response [7], as well as the stability of flexible rotors.

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*Half-Frequency Whirl - A special case of instability generally associated with self-acting journal bearings. This instability occurs when the journal speed reaches a critical value. The journal axis whirls at a frequency of one-half or nearly one-half of the journal speed in the same direction as the journal rotation. The motion of the journal of the journal axis can be either conical or cylindrical.

II. DERIVATION OF EQUATIONS TO BE SOLVED

The object of this section is to obtain the Reynolds equation in the form

$$L\left\{u\right\} = f,$$

where L is a linear operator, f is known function and u is the desired solution. We may then obtain our solution by Galerkin's method. Thus, we consider the nondimensional Reynolds equation

$$L\left\{p^2\right\} \equiv \left\{\frac{\partial}{\partial \theta}\left[h^3 \frac{\partial}{\partial \theta}\right] + \frac{\partial}{\partial z}\left[h^3 \frac{\partial}{\partial z}\right]\right\}\left\{p^2\right\} = 2\Lambda\left[\frac{\partial}{\partial \theta}(hp) + 2\frac{\partial}{\partial t}(hp)\right],$$

$$p^2\left(\pm \frac{\beta}{2}, z\right) = p^2\left(\theta, \pm L/D\right) = 1. \tag{1}$$

We make the following three assumptions:

(I) Λ is small (i.e., may be used as a perturbation parameter),

(II) p does not depend on time explicitly; i.e.,

$$\frac{\partial p}{\partial t} = \frac{\partial p}{\partial \epsilon}\dot{\epsilon} + \frac{\partial p}{\partial \dot{\epsilon}}\ddot{\epsilon} + \ldots + \frac{\partial p}{\partial \alpha}\dot{\alpha} + \frac{\partial p}{\partial \dot{\alpha}}\ddot{\alpha} + \ldots$$

(III) Time derivatives (e.g., $\dot{\epsilon}, \dot{\alpha}, \ddot{\epsilon}, \ddot{\alpha}$) are small enough so as to make their products negligible.

Then, using Assumption (I), we write

$$p(\theta,z) = \sum_{k=o}^{\infty} \Lambda^k p^{(k)}, \tag{2}$$

where

$$p^{(o)} \equiv 1.$$

Thus,

$$p^2(\theta,z) = \sum_{k=o}^{\infty} \Lambda^k \sum_{j=o}^{k} p^{(j)} p^{(k-j)} \tag{3}$$

Substitution of (2) and (3) in (1) gives

$$L\left\{p^2\right\} \equiv \left\{\frac{\partial}{\partial \theta}\left[h^3 \frac{\partial}{\partial \theta}\right] + \frac{\partial}{\partial z}\left[h^3 \frac{\partial}{\partial z}\right]\right\}\left\{\sum_{k=o}^{\infty} \Lambda^k \left[\sum_{j=o}^{k} p^{(j)} p^{(k-j)}\right]\right\} =$$

$$= 2\Lambda\left\{\frac{\partial}{\partial \theta} + 2\frac{\partial}{\partial t}\right\}\left\{h \sum_{k=o}^{\infty} \Lambda^k p^{(k)}\right\}, \tag{4}$$

from which, equating like powers of Λ, we obtain

$$\sum_{j=0}^{k} L\left\{p^{(j)}p^{(k-j)}\right\} = \sum_{j=0}^{k}\left\{\frac{\partial}{\partial\theta}\left[h^3\frac{\partial}{\partial\theta}(p^{(j)}p^{(k-j)})\right]+\frac{\partial}{\partial z}\left[h^3\frac{\partial}{\partial z}(p^{(j)}p^{(k-j)})\right]\right\}=$$

$$= 2\left\{\frac{\partial}{\partial\theta} + 2\frac{\partial}{\partial t}\right\} hp^{(k-1)}; \quad k = 1,2,... \tag{5}$$

Under Assumption (II) above, (5) becomes

$$\sum_{j=0}^{k} L\left\{p^{(j)}p^{(k-j)}\right\} = 2\left[\frac{\partial}{\partial\theta} + 2\sum_{r=1}^{k}(\epsilon^{[r]}\frac{\partial}{\partial\epsilon^{[r-1]}} + \alpha^{[r]}\frac{\partial}{\partial\alpha^{[r-1]}})\right](hp^{(k-1)}) \tag{6}$$

where

$$f^{[r]} = d^r f/dt^r$$

$$k = 1,2,...$$

We note then, that for $\underline{k = 1}$, since $p^{(0)} \equiv 1$, (6) becomes

$$L\left\{p^{(1)}\right\} = \left\{\frac{\partial}{\partial\theta}(h^3\frac{\partial}{\partial\theta}) + \frac{\partial}{\partial z}(h^3\frac{\partial}{\partial z})\right\}\left\{p^{(1)}\right\} = \frac{\partial h}{\partial\theta} + 2\left[\dot{\epsilon}\frac{\partial h}{\partial\epsilon} + \dot{\alpha}\frac{\partial h}{\partial\alpha}\right] . \tag{6a}$$

For $\underline{k>1}$, we transpose all terms containing $p^{(\ell)}$, with $\ell<k$, to the right side of the equation to obtain

$$L\left\{p^{(k)}\right\} = \left\{\frac{\partial}{\partial\theta} + 2\sum_{r=1}^{k}(\epsilon^{[r]}\frac{\partial}{\partial\epsilon^{[r-1]}} + \alpha^{[r]}\frac{\partial}{\partial\alpha^{[r-1]}})\right\}\left\{hp^{(k-1)}\right\}$$

$$- \tfrac{1}{2}\sum_{j=1}^{k-1} L\left\{p^{(j)}p^{(k-j)}\right\}, \quad k = 2,3,... \tag{7}$$

We now represent each perturbation pressure $p^{(k)}$ as the sum of its steady state component and its time-dependent components in the form:

$$p^{(k)} = p_{00}^{(k)} + \sum_{r=1}^{k}(\epsilon^{[r]}p_{r0}^{(k)} + \alpha^{[r]}p_{0r}^{(k)}) , \quad k = 1,2,... \tag{8}$$

(from (6a), we note that for k =1:

$$L\left\{p_{00}^{(1)}\right\} = \frac{\partial h}{\partial\theta} ; \; L\left\{p_{10}^{(1)}\right\} = 2\frac{\partial h}{\partial\epsilon} ; \; L\left\{p_{01}^{(1)}\right\} = 2\frac{\partial h}{\partial\alpha} . \;) \tag{9}$$

Using (8) and Assumption (III), we may write

$$p^{(j)}p^{(k-j)} = p_{00}^{(j)}\, p_{00}^{(k-j)} + p_{00}^{(k-j)} \sum_{r=1}^{j} (\epsilon^{[r]} p_{r0}^{(j)} + \alpha^{[r]} p_{0r}^{(j)})$$

$$+ p_{00}^{(j)} \sum_{r=1}^{k-j} (\epsilon^{[r]} p_{r0}^{(k-j)} + \alpha^{[r]} p_{0r}^{(k-j)}), \qquad (10)$$

for

$k = 2,3\ldots,$

$j = 1,2,\ldots,k.$

Then substitution of (8) and (10) in (7) yields

$$L\left\{p_{00}^{(k)} + \sum_{r=1}^{k} (\epsilon^{[r]} p_{r0}^{(k)} + \alpha^{[r]} p_{0r}^{(k)})\right\} =$$

$$= \frac{\partial}{\partial\theta}\left[hp_{00}^{(k-1)} + h\sum_{r=1}^{k-1}(\epsilon^{[r]} p_{r0}^{(k-1)} + \alpha^{[r]} p_{0r}^{(k-1)})\right] + 2\left[\dot{\epsilon}\,\frac{\partial}{\partial\epsilon}\,(hp_{00}^{(k-1)})\right.$$

$$\left. + \dot{\alpha}\,\frac{\partial}{\partial\alpha}\,(hp_{00}^{(k-1)})\right] + 2\sum_{r=2}^{k} (\epsilon^{[r]} hp_{r-1,0}^{(k-1)} + \alpha^{[r]} hp_{0,r-1}^{(k-1)})$$

$$-\tfrac{1}{2}\sum_{j=1}^{k-1} L\left\{p_{00}^{(j)} p_{00}^{(k-j)}\right\} - \tfrac{1}{2}\sum_{j=1}^{k-1} L\left\{p_{00}^{(k-j)} \sum_{r=1}^{j} (\epsilon^{[r]} p_{r0}^{(j)}) + \alpha^{[r]} p_{0r}^{(j)})\right\}$$

$$+ L\left\{p_{00}^{(j)} \sum_{r=1}^{k-j} (\epsilon^{[r]} p_{r0}^{(k-j)} + \alpha^{[r]} p_{0r}^{(k-j)})\right\}, \qquad (11)$$

for $k = 2,3,\ldots$

We note (from (1)) that L is linear; i.e., $L\{au + bv\} = aL\{u\} + bL\{v\}$.

However, L acting on a product is given by

$$L\{uv\} = uL\{v\} + vL\{u\} + 2h^3\left(\frac{\partial u}{\partial\theta}\,\frac{\partial v}{\partial\theta} + \frac{\partial u}{\partial z}\,\frac{\partial v}{\partial z}\right)$$

$$= uL\{v\} + vL\{u\} + 2M(u,v) \qquad (12)$$

We also now define the "known" functions $f(\theta,z)$ by

$$L\left\{p_{00}^{(k)}\right\} = f_{00}^{(k)},$$

$$L\left\{p_{r0}^{(k)}\right\} = f_{r0}^{(k)}, \qquad r = 1,2,\ldots,k, \qquad (13)$$

$$L\left\{p_{0r}^{(k)}\right\} = f_{0r}^{(k)},$$

noting that any $p^{(j)}$ occuring in an expression for $f^{(k)}$ has $j<k$.

A. Steady State Equations

Consideration of (11), and use of the relations (12) and (13) gives

$$L\{p_{00}^{(k)}\} = f_{00}^{(k)} = \frac{\partial}{\partial\theta}(hp_{00}^{(k-1)}) - \tfrac{1}{2}\sum_{j=1}^{k-1}(p_{00}^{(j)} f_{00}^{(k-j)} + p_{00}^{(k-j)} f_{00}^{(j)} + 2M(p_{00}^{(j)}, p_{00}^{(k-j)}) \quad , \tag{14}$$

for $k = 2,3,\ldots$

B. Time-Dependent Equations ($k = 2,3,\ldots$)

Again considering (11), matching like ϵ- and α- derivative terms, and using the relations (12) and (13) gives, after some manipulation:

$$L\{p_{10}^{(k)}\} = f_{10}^{(k)} = 2\frac{\partial}{\partial\epsilon}(hp_{00}^{(k-1)}) + \frac{\partial}{\partial\theta}(hp_{10}^{(k-1)}) - \sum_{r=1}^{k-1} p_{00}^{(r)} f_{10}^{(k-r)} + p_{10}^{(k-r)} f_{00}^{(r)} + 2M(p_{00}^{(r)}, p_{10}^{(k-r)})$$

$$L\{p_{j0}^{(k)}\} = f_{j0}^{(k)} = 2hp_{j-1,0}^{(k-1)} + \frac{\partial}{\partial 0}(hp_{j,0}^{(k-1)}) - \sum_{r=1}^{k-j}(p_{00}^{(r)} f_{j0}^{(k-r)} + p_{jo}^{(k-r)} f_{00}^{(r)} + 2M(p_{00}^{(r)}, p_{j0}^{(k-r)}) \quad , \qquad j = 1,2,3,\ldots, k-1 \tag{15}$$

$$L\{p_{k0}^{(k)}\} = f_{k0}^{(k)} = 2hp_{k-1,0}^{(k-1)} \quad ,$$

$$L\{p_{01}^{(k)}\} = f_{01}^{(k)} = 2\frac{\partial}{\partial\alpha}(hp_{00}^{(k-1)}) + \frac{\partial}{\partial\theta}(hp_{01}^{(k-1)}) - \sum_{r=1}^{k-1}(p_{00}^{(r)} f_{01}^{(k-r)} + p_{01}^{(k-r)} f_{00}^{(r)} + 2M(p_{00}^{(r)}, p_{01}^{(k-r)}) \quad ,$$

$$L\{p_{0j}^{(k)}\} = f_{0j}^{(k)} = 2hp_{0,j-1}^{(k-1)} + \frac{\partial}{\partial\theta}(hp_{0,j}^{(k-1)}) - \sum_{r=1}^{k-j}(p_{00}^{(r)} f_{0j}^{(k-r)} + p_{0j}^{(k-r)} f_{00}^{(r)} + 2M(p_{00}^{(r)}, p_{0j}^{(k-r)}) \quad , \qquad j = 2,3,\ldots,k-1,$$

$$L\{p_{0k}^{(k)}\} = f_{0k}^{(k)} = 2hp_{0,k-1}^{(k-1)}.$$

C. Static Stability Derivatives

We are also interested in obtaining equations for the derivatives of the steady state pressures with respect to ϵ and α. Therefore, we consider

$$L\left\{p_{00}^{(k)}\right\} = \frac{\partial}{\partial\theta}\left(h^3 \frac{\partial p_{00}}{\partial\theta}\right) + \frac{\partial}{\partial z}\left(h^3 \frac{\partial p_{00}^{(k)}}{\partial z}\right) = f_{00}^{(k)} .$$

We assume that:

(1) All necessary derivatives exist, and

(2) h is independent of z.

Then, differentiating the above expression by ϵ, we obtain

$$L\left\{\frac{\partial p_{00}^{(k)}}{\partial\epsilon}\right\} = f_{\epsilon}^{(k)} = \frac{\partial f_{00}^{(k)}}{\partial\epsilon} - 3\left(\frac{1}{h} f_{00}^{(k)} - h \frac{\partial h}{\partial\theta} \frac{\partial p_{00}^{(k)}}{\partial\theta}\right) \frac{\partial h}{\partial\epsilon} - 3h^2 \frac{\partial p_{00}^{(k)}}{\partial\theta} \frac{\partial^2 h}{\partial\epsilon\partial\theta} . \tag{16}$$

Similarly, a differentiation by α yields

$$L\left\{\frac{\partial p_{00}^{(k)}}{\partial\alpha}\right\} = f_{\alpha}^{(k)} = \frac{\partial f_{00}^{(k)}}{\partial\alpha} - 3\left(\frac{1}{h} f_{00}^{(k)} - h \frac{\partial h}{\partial\theta} \frac{\partial p_{00}^{(k)}}{\partial\theta}\right) \frac{\partial h}{\partial\alpha} - 3h^2 \frac{\partial p_{00}^{(k)}}{\partial\theta} \frac{\partial^2 h}{\partial\alpha\partial\theta} . \tag{17}$$

Thus, for any power k of the perturbation series (2) for p, we have obtained a set of partial differential equations for the steady state pressure, the time-dependent pressure components, and the derivatives of the steady-state pressure with respect to ϵ and α all of which are in the form

$$L\{u\} = f$$

suitable for numerical solution by the Galerkin method. We summarize these below for k = 1 and 2.

k = 1:

$$p^{(1)} = p_{00}^{(1)} + \dot{\epsilon} p_{10}^{(1)} + \dot{\alpha} p_{01}^{(1)} ,$$

where

$$L\left\{p_{00}^{(1)}\right\} = f_{00}^{(1)} = \frac{\partial h}{\partial\theta} ,$$

$$L\left\{p_{10}^{(1)}\right\} = f_{10}^{(1)} = 2\,\frac{\partial h}{\partial \epsilon}\,,$$

$$L\left\{p_{01}^{(1)}\right\} = f_{01}^{(1)} = 2\,\frac{\partial h}{\partial \alpha}\,, \tag{9}$$

and

$$L\left\{\frac{\partial p_{00}^{(1)}}{\partial \epsilon}\right\} = f_{\epsilon}^{(1)} = (1-3h^2\,\frac{\partial p_{00}^{(1)}}{\partial \theta})\frac{\partial^2 h}{\partial \epsilon \partial \theta} - \frac{3}{h}\,\frac{\partial h}{\partial \theta}\,\frac{\partial h}{\partial \epsilon}\,(1-h^2\,\frac{\partial p_{00}^{(1)}}{\partial \theta}), \tag{18}$$

$$L\left\{\frac{\partial p_{00}^{(1)}}{\partial \alpha}\right\} = f_{\alpha}^{(1)} = (1-3h^2\,\frac{\partial p_{00}}{\partial \theta})\frac{\partial^2 h}{\partial \alpha \partial \theta} - \frac{3}{h}\,\frac{\partial h}{\partial \theta}\,\frac{\partial h}{\partial \alpha}\,(1-h^2\,\frac{\partial p_{00}^{(1)}}{\partial \theta}). \tag{19}$$

<u>k = 2:</u>

$$p^{(2)} = p_{00}^{(2)} + \dot{\epsilon}\,p_{10}^{(2)} + \ddot{\epsilon}\,p_{20}^{(2)} + \dot{\alpha}\,p_{01}^{(2)} + \ddot{\alpha}\,p_{02}^{(2)}\,,$$

where

$$\left.\begin{aligned} L\left\{p_{00}^{(2)}\right\} = f_{00}^{(2)} &= \frac{\partial}{\partial \theta}\,(hp_{00}^{(1)}) - p_{00}^{(1)}f_{00}^{(1)} - M(p_{00}^{(1)},p_{00}^{(1)}) \\ &= h\,\frac{\partial p_{00}^{(1)}}{\partial \theta} - h^3\left[\left(\frac{\partial p_{00}^{(1)}}{\partial \theta}\right)^2 + \left(\frac{\partial p_{00}^{(1)}}{\partial z}\right)^2\right]\,, \end{aligned}\right\} \tag{20}$$

$$L\left\{p_{10}^{(2)}\right\} = f_{10}^{(2)} = \frac{\partial}{\partial \theta}\,(hp_{10}^{(1)}) + 2\,\frac{\partial}{\partial \epsilon}(hp_{00}^{(1)}) - p_{00}^{(1)}f_{10}^{(1)}$$

$$-p_{10}^{(1)}f_{00}^{(1)} - 2M(p_{00}^{(1)},p_{10}^{(1)}), \tag{21}$$

$$L\left\{p_{20}^{(2)}\right\} = f_{20}^{(2)} = 2hp_{10}^{(1)} \tag{22}$$

$$L\left\{p_{01}^{(2)}\right\} = f_{01}^{(2)} = \frac{\partial}{\partial \theta}\,(hp_{01}^{(1)}) + 2\frac{\partial}{\partial \alpha}\,(hp_{00}^{(1)}) - p_{00}^{(1)}f_{01}^{(1)}$$

$$-p_{01}^{(1)}f_{00}^{(1)} - 2M\,(p_{00}^{(1)},p_{01}^{(1)}), \tag{23}$$

$$L\left\{p_{02}^{(2)}\right\} = f_{02}^{(2)} = 2hp_{01}^{(1)}, \tag{24}$$

and

$$L\left\{\frac{\partial p_{00}^{(2)}}{\partial \epsilon}\right\} = f_{\epsilon}^{(2)} = \frac{\partial f_{00}^{(2)}}{\partial \epsilon} - 3\left[\frac{1}{h} f_{00}^{(2)} - h \frac{\partial h}{\partial \theta} \frac{\partial p_{00}^{(2)}}{\partial \theta}\right] \frac{\partial h}{\partial \epsilon}$$

$$-3h^2 \frac{\partial p_{00}^{(2)}}{\partial \theta} \frac{\partial^2 h}{\partial \epsilon \partial \theta}, \tag{25}$$

$$L\left\{\frac{\partial p_{00}^{(2)}}{\partial \alpha}\right\} = f_{\alpha}^{(2)} = \frac{f_{00}^{(2)}}{\partial \alpha} - 3\left[\frac{1}{h} f_{00}^{(2)} - h \frac{\partial h}{\partial \theta} \frac{\partial p_{00}^{(2)}}{\partial \theta}\right] \frac{\partial h}{\partial \alpha}$$

$$-3h^2 \frac{\partial p_{00}^{(2)}}{\partial \theta} \frac{\partial^2 h}{\partial \alpha \partial \theta} \tag{26}$$

III. SOLUTION BY GALERKIN'S METHOD

In the present report, we outline briefly the steps followed in solving our equations by Galerkin's method. This method is equivalent to the Rayligy-Ritz procedure in any problem where a minimum principle applies, as in this case. A complete discussion is given in [2].

We consider the partial differential equation

$$L\{u\} = \frac{\partial}{\partial\theta}\left(h^3 \frac{\partial u}{\partial\theta}\right) + \frac{\partial}{\partial z}\left(h^3 \frac{\partial u}{\partial z}\right) = f(\theta,z). \tag{27}$$

We assume that (See Figure 1.)

$$h = 1-\epsilon \cos(\theta-\alpha). \tag{28}$$

As boundary conditions, we have

$$u\left(\pm\frac{\beta}{2}, z\right) = u\left(\theta, \pm\frac{L}{D}\right) = 0. \tag{29}$$

From the above it follows that u is an even function of z, i.e.,

$$f(\theta,z) = f(\theta,-z) \tag{30}$$

We assume an approximate solution $\bar{u}(\theta,z)$ of the form

$$\bar{u}(\theta,z) = \sum_{n=1}^{N} Z_n(z) \sum_{m=1}^{M} c_{mn} \Theta_m(\theta), \tag{31}$$

where the functions $Z_n(z)$ and $\Theta_m(\theta)$ are linearly independent elements of a complete set (i.e., a set complete in the sense of the Weierstrass approximation theorem [3], and satisfy the

boundary conditions (29). In our case, we choose

$$Z_n(z) = \cos\left[\frac{(2n-1)}{2}\ \frac{\pi D}{L}\ z\right],$$

$$\Theta_m(\theta) = \sin\left[\frac{m\pi}{\beta}\ \theta + \frac{m\pi}{2}\right]. \tag{32}$$

The c_{mn} are undetermined coefficients.

Then, following the procedure outlined in [2], we determine the coefficients c_{mn} from

$$2\int_{\beta/2}^{\beta/2}\int_0^{L/D}\left[L\left\{\bar{u}\right\}-f\right] Z_\nu(z)\Theta_\mu(\theta)dz\ d\theta = 0, \tag{33a}$$

for $\nu = 1,2,\ldots,N$; $\mu = 1,2,\ldots,M$. Or,

$$\sum_{n=1}^{N}\sum_{m=1}^{M}\int_{-\beta/z}^{\beta/z}\int_0^{L/D}\left[Z_n\frac{d}{d\theta}\left(h^3\frac{d\Theta_m}{d\theta}\right) + h^3\Theta_m\ \frac{d^2Z_n}{dz^2}\right] Z_\nu\Theta_\mu\ dz\ d\theta =$$

$$= \int_{\beta/2}^{\beta/2}\Theta_\mu\int_0^{L/D} f(\theta,z)Z_\nu\ dz\ d\theta, \tag{33b}$$

for $\nu = 1,2,\ldots,N$; $\mu = 1,2,\ldots,M$.

Ordinarily, to obtain the coefficients c_{mn}, we would have to invert a matrix of order MN x MN. However, with our choice of $Z_n(z)$ and the assumption on h (Eq. 28), the orthogonality of the cosine functions requires that $n = \nu$, and (33b) becomes partially uncoupled. Therefore, we have instead N matrices each of order M x M to invert.

In the preceding section, we derived the forcing terms f corresponding to each pressure component and the steady state pressure derivatives with respect to ϵ and α. We now correspond to each of these a set of coefficient matrices:

$$f_{rs}^{(k)} \longleftrightarrow p_{rs}^{(k)} = \sum_{n=1}^{N}\sum_{m=1}^{M} c_{\substack{mn\\rs}}^{(k)}\Theta_m(\theta)\,Z_n(z), \tag{34a}$$

$$f_\epsilon^{(k)} \longleftrightarrow \frac{\partial p_{00}^{(k)}}{\partial\epsilon} = \sum_{n=1}^{N}\sum_{m=1}^{M} c_{\substack{mn\\\epsilon}}^{(k)}\Theta_m(\theta)\,Z_n(z), \tag{34b}$$

$$f_\alpha^{(k)} \longleftrightarrow \frac{\partial p_{00}^{(k)}}{\partial \alpha} = \sum_{n=1}^{N} \sum_{m=1}^{M} c_{mn\,\alpha}^{(k)} \Theta_m(\theta)\, Z_n(z). \tag{34c}$$

The uncoupling in z described above permits us to write the systems to be inverted as follows, for each value of n (n =1, 2,...,N):

$$\sum_{m=1}^{M} c_{mn\,rs}^{(k)} A_{\mu m;n} = \Phi_{\mu n\,rs}^{(k)} = -\frac{2\pi D}{\beta L} \int_{\beta/2}^{\beta/2} \Theta_\mu \int_0^{L/D} f_{rs}^{(k)} Z_n \, dz \, d\theta;$$

$$\mu = 1,2,\ldots,M, \tag{35a}$$

$$\sum_{m=1}^{M} c_{mn\,\epsilon}^{(k)} A_{\mu m;n} = \Phi_{\mu n\,\epsilon}^{(k)} = -\frac{2\pi D}{\beta L} \int_{-\beta/2}^{\beta/2} \Theta_\mu \int_0^{L/D} f_{\epsilon}^{(k)} Z_n \, dz \, d\theta;$$

$$\mu = 1,2,\ldots,M, \tag{35b}$$

$$\sum_{m=1}^{M} c_{mn\,\alpha}^{(k)} A_{\mu m;n} = \Phi_{\mu n\,\alpha}^{(k)} = -\frac{2\pi D}{\beta L} \int_{-\beta/2}^{\beta/2} \Theta_\mu \int_0^{L/D} f_{\epsilon}^{(k)} Z_n \, dz \, d\theta;$$

$$\mu = 1,2,\ldots M. \tag{35c}$$

In each of the above, the elements of the matrices $A_{\mu m;n}$ are identical, and are given by

$$A_{\mu m;n} = \begin{cases} a_0 (\kappa_m^2 + \lambda_n^2) + \kappa_m^2 \sum_{j=1}^{3} \left\{ a_j \cos j\alpha \left[1 - \frac{2(\kappa_m^2 + \lambda_n^2)}{j^2} \right] \times \right. \\ \qquad \left. \times \left[\frac{\sin (j\beta/2)}{(\frac{j\beta}{\pi})^2 - 4m^2} \right] \right\}, \quad m = \mu \\ -m\mu \sum_{j=1}^{3} \left\{ a_j \sin j\alpha \left[\kappa_m^2 + \kappa_\mu^2 + 2\lambda_n^2 - j^2 \right] \left[\frac{\cos (j\beta/2)}{\left[(\frac{j\beta}{\pi})^2 - m^2 - \mu^2 \right]^2 - (2m\mu)^2} \right] \right\}, \\ \qquad m \neq \mu \\ \qquad m + \mu \text{ odd} \\ -m\mu \sum_{j=1}^{3} \left\{ a_j \cos j\alpha \left[\kappa_m^2 + \kappa_\mu^2 + 2\lambda_n^2 - j^2 \right] \left[\frac{\sin (j\beta/2)}{\left[(\frac{j\beta}{\pi})^2 - m^2 - \mu^2 \right]^2 - (2m\mu)^2} \right] \right\}, \\ \qquad m \neq \mu \\ \qquad m + \mu \text{ even} \end{cases} \tag{36}$$

where

$$a_0 = \frac{\pi}{2}\left(1 + \frac{3}{2}\epsilon^2\right),$$

$$a_1 = -\frac{6\beta}{\pi}\epsilon\left(1 + \frac{\epsilon^2}{4}\right),$$

$$a_2 = -\frac{6\beta}{\pi}\epsilon^2,$$

$$a_3 = -\frac{3}{2}\frac{\beta}{\pi}\epsilon^3,$$

$$\kappa_m = m\pi/\beta,$$

$$\lambda_n = \left(\frac{2n-1}{2}\right)\frac{\pi D}{L}.$$

Note that the $A^{\cdot}_{\mu m;n}$ matrices are symmetric in μ and m. They also contain some removable singularities. The limiting values of terms in (36) are given as follows:

$$\left.\begin{aligned}
&\lim_{x \to 2m}\left\{\frac{\sin(\pi x/2)}{x^2 - 4m^2}\right\} = \frac{(-)^m\pi}{8m}\ ;\quad \mu = m \\
&\lim_{x \to |\mu \pm m|}\left\{\frac{\cos(\pi x/2)}{x^2-m^2-\mu^2)^2-(2m\mu)^2}\right\} = \mp\frac{\pi \sin\left[\frac{\pi}{2}|\mu \pm m|\right]}{16\,\mu m\,|\mu \pm m|}\ ;\quad \left\{\begin{matrix}\mu > m \\ \mu + m \text{ odd}\end{matrix}\right\} \\
&\lim_{x \to |\mu \pm m|}\left\{\frac{\sin(\pi x/2)}{(x^2-m^2-\mu^2)^2-(2m\mu)^2}\right\} = \pm\frac{\pi \cos\left[\frac{\pi}{2}|\mu \pm m|\right]}{16\,\mu m\,|\mu \pm m|}\ ;\quad \left\{\begin{matrix}\mu > m \\ \mu + m \text{ even}\end{matrix}\right\}
\end{aligned}\right\} \quad (37)$$

The computational procedure to obtain pressure is then as follows:

1. Calculate the $A_{\mu m;n}$ matrices from (36) and invert them.
2. For the appropriate pressure component obtain the forceing term from Eqs. (9), (14)-(17). These are given in more detail for k = 1,2 in Eqs. (9), (18)-(26) on pages

3. Obtain the matrix of Φ_n vectors from Eqs. (35).
4. Obtain the coefficients c_{mn} by multiplying the Φ_n vectors by the corresponding inverses of the $A_{\mu m;n}$ matrices.
5. The pressure component is then found using Eqs. (34), (8) and (2).

IV. CALCULATION OF FORCES

The dimensionless forces along and perpendicular to the line of centers are denoted F_R and F_T respectively, and are given by

$$F_R = 2 \int_{-\beta/2}^{\beta/2} \cos(\theta-\alpha) \int_0^{L/D} p \, dz \, d\theta, \tag{38a}$$

$$F_T = -2 \int_{-\beta/2}^{\beta/2} \sin(\theta-\alpha) \int_0^{L/D} p \, dz \, d\theta, \tag{38b}$$

Where p is the pressure component or its derivative with respect to ϵ or α. Substituting the appropriate summation

$$p = \sum_{n=1}^{N} \sum_{m=1}^{M} c_{mn} \, \Theta_m \, Z_n$$

in (38), we may then integrate directly to obtain

$$F_R = \frac{8}{\pi} \frac{L}{D} \frac{\beta}{\pi} \left\{ \sum_{n=1}^{N} \frac{(-)^n}{2n-1} \sum_{m=1}^{M} c_{mn} \left[\frac{m}{m^2-(\beta/\pi)^2} \right] \times \right.$$

$$\left. \times \begin{bmatrix} -\cos\frac{\beta}{2} \cos\alpha, & (m \text{ odd}) \\ \sin\frac{\beta}{2} \sin\alpha, & (m \text{ even}) \end{bmatrix} \right\}, \tag{39a}$$

$$F_T = \frac{-8}{\pi} \frac{L}{D} \frac{\beta}{\pi} \left\{ \sum_{n=1}^{N} \frac{(-)^n}{2n-1} \sum_{m=1}^{M} c_{mn} \left[\frac{m}{m^2-\beta/\pi)^2} \right] \times \right.$$

$$\left. \times \begin{bmatrix} \cos(\beta/2)\sin\alpha, & (m \text{ odd}) \\ \sin(\beta/2)\cos\alpha, & (m \text{ even}) \end{bmatrix} \right\}. \tag{39b}$$

V. EXTRAPOLATION PROCEDURE

By use of Richardson's extrapolation procedure (see e.g. [4]), and Tanner's use * of extrapolation in [1], we may use results computed with different values of M and/or N to obtain a closer approximation to the true solution. We will assume that the z dependence of our solution can be obtained with very few functions Z_n. Henceforth, we consider N fixed. Then, if we use two values of M, we obtain the extrapolation formula

$$F = (M_2^2 \ E_2 - M_1^2 \ E_1) \ / \ (M_2^2 - M_1^2), \tag{40}$$

where

F = extrapolated result,

E_i = computed result for $M = M_i$; $i = 1,2$.

It is of interest to include a short discussion of error growth in the calculation of $f^{(k)}$ for successive values of k. We assume truncation error per calculation fixed. (That is, we assume M and N are fixed. Obviously, the greater M and N to begin with, the lower the truncation error, from the Weierstrass theorem). Thus, for a given k, we have

$$p_*^{(k)} = p^{(k)} + \epsilon^{(k)},$$

where

$p_*^{(k)}$ = calculated pressure at the $k^{\underline{th}}$ stage of computation,

$p^{(k)}$ = true pressure at the $k^{\underline{th}}$ stage,

$\epsilon^{(k)}$ = error at the $k^{\underline{th}}$ stage.

To calculate $f^{(k+1)}$ requires derivatives of $p^{(k)}$. But, in [2], it is shown that the errors in the first derivatives of the solution function are of the same order as those in the solution

---

* In [1], Tanner used a three-term extrapolation, wherein the first term obviously was not good. If he had used his two better approximate values in an extrapolation of the form (40) above, his results would have been much closer to those he used as a standard.

function itself. Thus, it can be seen that while error does increase with k, the process is essentially stable in that all errors in the (k + 1) stage are of order $\epsilon^{(k)}$, and therefore, higher values of k can be computed if the initial truncation errors are kept low.

VI. SAMPLE RESULTS

A digital computer program has been written to perform the calculations indicated above for k = 1. Currently being checked out is a subroutine to extend these results to the case k = 2.

Some computer program results are given herein. The physical inputs were as follows:

Shoe angle $\beta = 120^{\circ}$

Angle of line of centers $\alpha = 60^{\circ}$

L/D = 1.0

Eccentricity ration ϵ = 0.01, 0.8

Results were computed with N = 2, and with two values of M: 4 and 6. A tabulation follows:

Force Component	ϵ = 0.01			ϵ = 0.8		
	M			M		
	4	6	$\overset{\infty}{\text{(Extrapolation)}}$	4	6	$\overset{\infty}{\text{(Extrapolation)}}$
$F_{R_{00}}^{(1)} = -\frac{1}{2} F_{R_{01}}^{(1)}$	0.00187	0.00187	0.00187	1.367	1.490	1.589
$F_{T_{00}}^{(1)} = -\frac{1}{2} F_{T_{01}}^{(1)}$	0.00384	0.00384	0.00384	1.111	1.140	1.164
$F_{R_{10}}^{(1)}$	0.3453	0.3467	0.3479	5.533	6.865	7.931
$F_{T_{10}}^{(1)}$	0.3732	0.3730	0.3728	3.417	3.726	3.973
$\frac{\partial}{\partial\epsilon} F_{R_{oo}}^{(1)}$	0.1899	0.1898	0.1897	9.397	11.081	12.428
$\frac{\partial}{\partial\epsilon} F_{T_{00}}^{(1)}$	0.3869	0.3874	0.3878	5.569	5.859	6.091
$\frac{\partial}{\partial\alpha} F_{R_{00}}^{(1)}$	0.00169	0.00170	0.00171	-2.030	-1.693	-1.423
$\frac{\partial}{\partial\alpha} F_{T_{00}}^{(1)}$	0.00178	0.00177	0.00176	-1.861	-1.807	-1.764

A plot of the steady state forces and pressure profile is given as Figure 1. The above results have been compared with those obtained in [5], and good agreement was found.

CONCLUSIONS

1. The approach described herein provides a fast, inexpensive, accurate method of obtaining partial arc and pivoted shoe bearing data.
2. It operates over a wide range of conditions (including high values of ϵ), over which difficulty is encountered in other (e.g., iterative) approaches.
3. The expressions given in the present report include compressibility effects of all orders (i.e., powers of Λ). The computer program can be extended to obtain these higher order effects.

RECOMMENDATIONS

A. Computer Program

It is recommended that:

1. The computer program be extended to include second-order effects (k = 2).
2. The program be modified so that Richardson's extrapolation procedure is performed automatically by the computer.
3. Programming be included so as to perform numerical quadratures for higher compressibility effects (values of k), since analytical procedures are not practical.

B. Applications

1. The computer program should be used to generate design charts for partial arc gas bearings at high ambients to check and supplement presently available data.
2. The dynamic force derivatives calculated by this computer program can be incorporated into rotor dynamics analysis to study critical speeds and frequency response.
3. The static and dynamic force derivatives can be used to calculate the threshold speed of instability of rotors supported in partial arc bearings.
4. This computer program can be used as a subroutine in a larger program to study static and dynamic characteristics of multilobe and pivoted shoe bearings.

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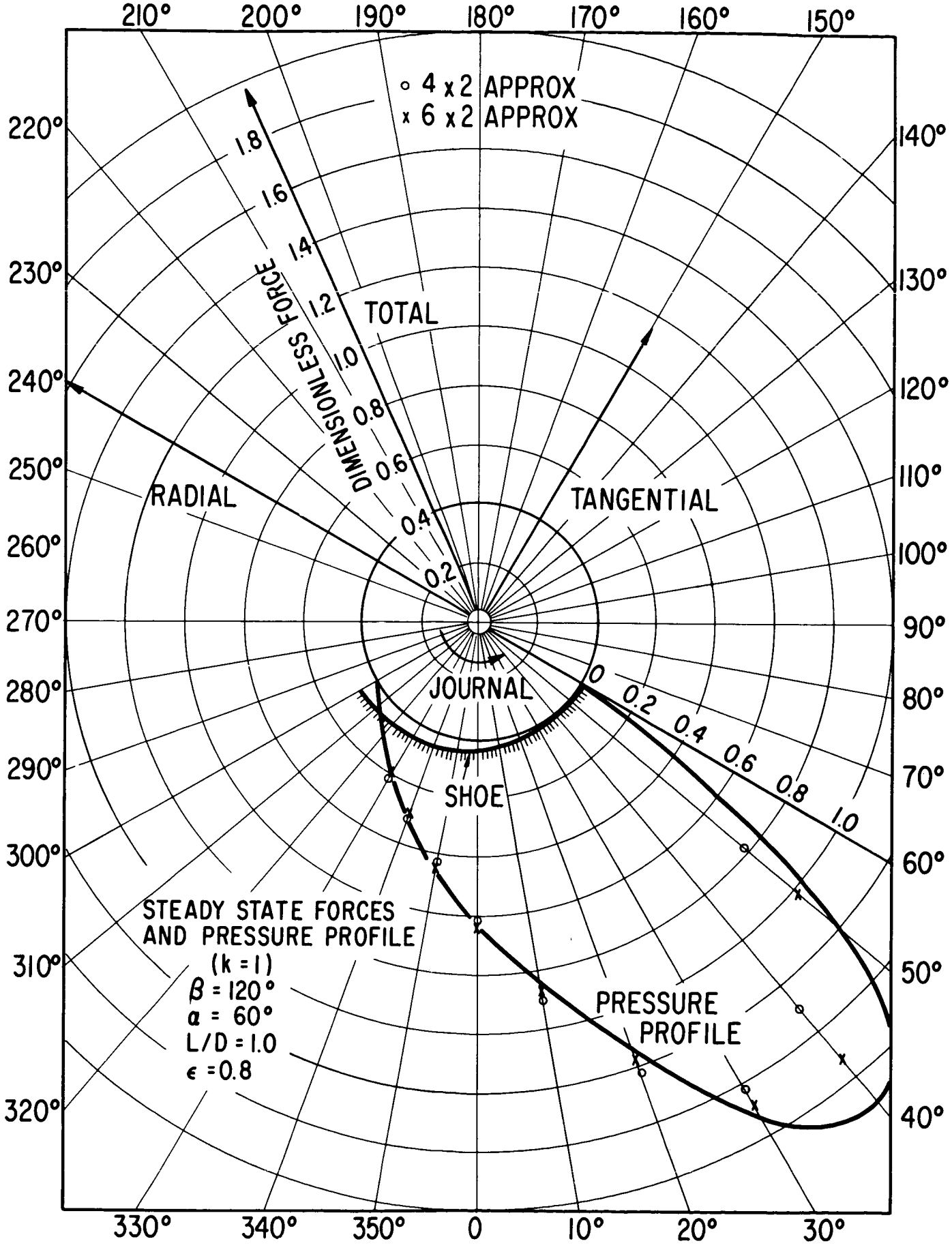

FIGURE 1
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